

H.SProblem Sheet-7

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TRIGONOMETRYMULTIPLE ANGLES

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1. Show that $\sin^3 \alpha + \sin^3(120^\circ + \alpha) + \sin^3(240^\circ + \alpha) = -\frac{3}{4} \sin 3\alpha$
2. Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$.
3. Show that $32 \sin^6 \theta = 10 - 15 \cos 2\theta + 6 \cos 4\theta - \cos 6\theta$
4. Prove that, $16 \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} = 1$.
5. $\sin^2(\alpha + \alpha) + \sin^2(\alpha + \beta) - 2 \cos(\alpha - \beta) \sin(\alpha + \alpha) \sin(\alpha + \beta)$ is independent of α .
6. If $13\theta = \pi$, Prove that $\cos \theta \cos 2\theta \cos 3\theta \cos 4\theta \cos 5\theta \cos 6\theta = \frac{1}{2}$
7. If $\tan \alpha + \tan \beta = \frac{\tan \alpha + \tan \beta}{1 + \tan \alpha \tan \beta}$ show that, $\sin 2\theta = \frac{\sin 2\alpha + \sin 2\beta}{1 + \sin 2\alpha \sin 2\beta}$
8. $(1 + \cos \frac{\pi}{8})(1 + \cos \frac{3\pi}{8})(1 + \cos \frac{5\pi}{8})(1 + \cos \frac{7\pi}{8}) = \frac{1}{8}$
9. Show that, $\frac{\sin \theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2} (\tan 27\theta - \tan \theta)$
10. $16 \cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} = -1$
11. If $\sin \alpha + \cos \alpha = \sqrt{2} \cos \alpha$ show that $\tan 2\alpha = 1$.
12. If $\tan \theta = \frac{1}{7}$ & $\tan \phi = \frac{1}{3}$ show that $\cos 2\theta = \sin 4\phi$.
13. (i) If $2 \cos \theta = a + \frac{1}{a}$ show that $\cos 2\theta = \frac{1}{2} (a^2 + \frac{1}{a^2})$
(ii) If $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$ prove that $2 \cos^2 \theta = 1 + \sin 2\alpha$.
14. If $\tan^2 \theta = 1 + 2 \tan^2 \phi$ show that, $\cos 2\theta + \sin^2 \phi = 0$.
15. If $\tan x = \frac{b}{a}$ find the value of $a^2 \csc 2x + b^2 \sec 2x$
16. If $2 \tan \alpha = 3 \tan \beta$ show that, $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$
17. If $\tan \theta = \sec 2\alpha$ show that, $\sin 2\theta = \frac{1 - \tan^4 \alpha}{1 + \tan^4 \alpha}$
18. If $\frac{\pi}{2} < \theta < \pi$ & $5 \sin^2 \theta + 3 \cos^2 \theta = 4$, find the values of $\sin 2\theta$ & $\cos 3\theta$.

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19. If $x^2 + y^2 = 1$ show that, $(3x - 4x^3)^2 + (3y - 4y^3)^2 = 1$

20. Prove that (i) $8\cos^4 \theta = 3 + 4\cos 2\theta + \cos 4\theta$

(ii) $8\sin^4 \theta = 3 - 4\cos 2\theta + \cos 4\theta$

(iii) $64\cos^3 \theta \cdot \sin^4 \theta = \cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos \theta$.

(iv) $\sin^2 A \cos^4 A = \frac{1}{16} + \frac{1}{32} \cos 2A - \frac{1}{16} \cos 4A - \frac{1}{32} \cos 6A$.

21. Show that, $\sec x = \frac{2}{\sqrt{2 + \sqrt{2 + 2\cos 4x}}}$

22. Using the formula $\tan A = \frac{\sin 2A}{1 + \cos 2A}$ find the values of $\tan 75^\circ$ & $\cot 22^\circ 30'$.

23. If $\tan(A+B) = 3\tan A$, prove that,

$$\sin(2A+2B) + \sin 2A = 2\sin 2B.$$

24. Prove that, $(2\cos \theta - 1)(2\cos 2\theta - 1)(2\cos 2^2 \theta - 1) \dots (2\cos 2^{n-1} \theta - 1)$
 $= \frac{2\cos 2^n \theta + 1}{2\cos \theta + 1}$

25. Show that,

$$(x \operatorname{cosec} \alpha + y \cot \alpha)(x \cot \alpha + y \operatorname{cosec} \alpha) = (x+y)^2 + 4xy \cot^2 \alpha.$$

26. If $a\sin \theta + b\cos \theta = c$ & $a \operatorname{cosec} \theta + b \sec \theta = c$, show that,

$$\sin 2\theta = \frac{2ab}{c^2 - a^2 - b^2}$$

27. If $x - \frac{1}{x} = 2i \sin \theta$ show that $x^4 - \frac{1}{x^4} = 2i \sin 4\theta$.

28. If $\tan B = \frac{\tan \theta_1 + \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$ show that

$$\sin 2B = \frac{\sin 2\theta_1 + \sin 2\theta_2}{1 + \sin 2\theta_1 \sin 2\theta_2}$$

29. If $\tan^2 x + 2\tan x \cdot \tan 2y = \tan^2 y + 2\tan y \tan 2x$; show that each side $\equiv 1$ or else $\tan x = \pm \tan y$.

30. $\frac{\tan(\alpha + \beta - \gamma)}{\tan(\alpha - \beta + \gamma)} = \frac{\tan \gamma}{\tan \beta}$, prove that $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$.